FIRST THEOREMS OF PROPOSITIONAL CALCULUS

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Abstract. This module includes first proofs of propositional calculus theorems.

Contents

module specification

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This module has the following specification:

Name:	proptheo1
Version:	1.00.00
Rule version:	1.00.00
Orgin:	http://www.meyling.com/principia/0_00_51/proptheo1_1.00.00_1.00.00.qedeq

The following modules were used:

Name:	propaxiom
Version:	1.00.00
Rule version:	1.00.00
Orgin:	propaxiom_1.00.00_1.00.00.qedeq
pdf:	propaxiom_1.00.00_1.00.00.pdf

First we prove a well known tautology:

Theorem 0.1.

$$(\neg P \lor P)$$

Proof.

1	$((P \Rightarrow Q) \Rightarrow ((A \lor P) \Rightarrow (A \lor Q)))$	add axiom axiom4
2	$(((P \lor P) \Rightarrow Q) \Rightarrow ((A \lor (P \lor P)) \Rightarrow (A \lor Q)))$	replace P by $(P \lor P)$ in 1
3	$(((P \lor P) \Rightarrow P) \Rightarrow ((A \lor (P \lor P)) \Rightarrow (A \lor P)))$	replace Q by P in 2
4	$(((P \lor P) \Rightarrow P) \Rightarrow ((\neg P \lor (P \lor P)) \Rightarrow (\neg P \lor P)))$	replace A by $\neg P$ in 3

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5	$((P \lor P) \Rightarrow P)$	add axiom axiom1
6	$((\neg P \lor (P \lor P)) \Rightarrow (\neg P \lor P))$	MP with 5, 4
7	$((P \Rightarrow (P \lor P)) \Rightarrow (\neg P \lor P))$	reverse abbreviation impl in 6 at occurence 1
8	$(P \Rightarrow (P \lor Q))$	add axiom axiom2
9	$(P \Rightarrow (P \lor P))$	replace Q by P in 8
10	$(\neg P \lor P)$	MP with 9, 7

□

We just use our first sentence to get the second theorem:

Theorem 0.2.

$$(P \Rightarrow P)$$

Proof.

1	$(\neg P \lor P)$	add sentence theo1
2	$(P \Rightarrow P)$	reverse abbreviation impl in 1 at occurence 1

□

And another use of the first theorem, to get the law of the excluded middle (tertium non datur):

Theorem 0.3.

$$(P \lor \neg P)$$

Proof.

1	$(\neg P \lor P)$	add sentence theo1
2	$((P \lor Q) \Rightarrow (Q \lor P))$	add axiom axiom3
3	$((\neg P \lor Q) \Rightarrow (Q \lor \neg P))$	replace P by $\neg P$ in 2
4	$((\neg P \lor P) \Rightarrow (P \lor \neg P))$	replace Q by P in 3
5	$(P \lor \neg P)$	MP with 1, 4

□

Also trivial is:

Theorem 0.4.

$$(P \Rightarrow \neg\neg P)$$

Proof.

1	$(P \lor \neg P)$	add sentence theo3
2	$(\neg P \lor \neg\neg P)$	replace P by $\neg P$ in 1
3	$(P \Rightarrow \neg\neg P)$	reverse abbreviation impl in 2 at occurence 1

□

Three negations:

Theorem 0.5.

$$(P \lor \neg\neg\neg P)$$

Proof.

1	$(P \Rightarrow \neg\neg P)$	add sentence theo4
2	$((P \Rightarrow Q) \Rightarrow ((A \lor P) \Rightarrow (A \lor Q)))$	add axiom axiom4
3	$((P \Rightarrow \neg\neg P) \Rightarrow ((A \lor P) \Rightarrow (A \lor \neg\neg P)))$	replace Q by $\neg\neg P$ in 2
4	$((A \lor P) \Rightarrow (A \lor \neg\neg P))$	MP with 1, 3
5	$((A \lor \neg P) \Rightarrow (A \lor \neg\neg\neg P))$	replace P by $\neg P$ in 4
6	$((P \lor \neg P) \Rightarrow (P \lor \neg\neg\neg P))$	replace A by P in 5
7	$(P \lor \neg P)$	add sentence theo3
8	$(P \lor \neg\neg\neg P)$	MP with 7, 6

□

Now we could prove the reverse of Proposition 4:

Theorem 0.6.

$$(\neg\neg P \Rightarrow P)$$

Proof.

1	$(P \lor \neg\neg\neg P)$	add sentence theo5
2	$((P \lor Q) \Rightarrow (Q \lor P))$	add axiom axiom3
3	$((P \lor \neg\neg\neg P) \Rightarrow (\neg\neg\neg P \lor P))$	replace Q by $\neg\neg\neg P$ in 2
4	$(\neg\neg\neg P \lor P)$	MP with 1, 3
5	$(\neg\neg P \Rightarrow P)$	reverse abbreviation impl in 4 at occurence 1

□

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