SOME MORE THEOREMS OF PREDICATE CALCULUS

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Abstract. This module includes first proofs of predicate calculus theorems.

Contents

module specification

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This module has the following specification:

Name:	predtheo2
Version:	1.00.00
Rule version:	1.02.00
Orgin:	`http://www.meyling.com/principia/0_00_51/predtheo2_1.00.00_1.02.00.qedeq`

The following modules were used:

Name:	predaxiom
Version:	1.00.00
Rule version:	1.00.00
Orgin:	`predaxiom_1.00.00_1.00.00.qedeq`
pdf:	`predaxiom_1.00.00_1.00.00.pdf`
Name:	prophilbert3
Version:	1.00.00
Rule version:	1.02.00
Orgin:	`prophilbert3_1.00.00_1.02.00.qedeq`
pdf:	`prophilbert3_1.00.00_1.02.00.pdf`

A simple implication:

Theorem 0.1.

$$(\exists x(R(x)) \ \Rightarrow\ \forall x(R(x)))$$

Proof.

Date: 2002-07-27T21:00:49.

1	$(\exists x(R(x)) \;\Rightarrow\; R(y))$	add axiom axiom5
2	$(R(y) \;\Rightarrow\; \forall x(R(x)))$	add axiom axiom6
3	$(\exists x(R(x)) \;\Rightarrow\; \forall x(R(x)))$	HS with 1 and 2

□

A well known implication:

Theorem 0.2.

$$(\forall x(R(x)) \;\Rightarrow\; \neg\exists x(\neg R(x)))$$

Proof.

1	$(\exists x(R(x)) \;\Rightarrow\; R(y))$	add axiom axiom5
2	$(\exists x(\neg R(x)) \;\Rightarrow\; \neg R(y))$	replace $R(@S_1)$ by $\neg R(@S_1)$ in 1
3	$(\neg\exists x(\neg R(x)) \;\vee\; \neg R(y))$	use abbreviation impl in 2 at occurence 1
4	$((P \;\vee\; Q) \;\Rightarrow\; (Q \;\vee\; P))$	add axiom axiom3
5	$((\neg\exists x(\neg R(x)) \;\vee\; Q) \;\Rightarrow\; (Q \;\vee\; \neg\exists x(\neg R(x))))$	replace P by $\neg\exists x(\neg R(x))$ in 4
6	$((\neg\exists x(\neg R(x)) \;\vee\; \neg R(y)) \;\Rightarrow\; (\neg R(y) \;\vee\; \neg\exists x(\neg R(x))))$	replace Q by $\neg R(y)$ in 5
7	$(\neg R(y) \;\vee\; \neg\exists x(\neg R(x)))$	MP with 3, 6
8	$(R(y) \;\Rightarrow\; \neg\exists x(\neg R(x)))$	reverse abbreviation impl in 7 at occurence 1
9	$(\forall y(R(y)) \;\Rightarrow\; \neg\exists x(\neg R(x)))$	Particularize by y in 8
10	$(\forall x(R(x)) \;\Rightarrow\; \neg\exists x(\neg R(x)))$	rename y into x in 9 at occurence 1

□

The reverse is also true:

Theorem 0.3.

$$(\neg\exists x(\neg R(x)) \;\Rightarrow\; \forall x(R(x)))$$

Proof.

1	$(R(y) \;\Rightarrow\; \forall x(R(x)))$	add axiom axiom6
2	$(\neg\forall x(R(x)) \;\Rightarrow\; \neg R(y))$	apply hilb7 in 1
3	$(\neg\forall x(R(x)) \;\Rightarrow\; \exists y(\neg R(y)))$	Generalize by y in 2
4	$(\neg\exists y(\neg R(y)) \;\Rightarrow\; \neg\neg\forall x(R(x)))$	apply hilb7 in 3
5	$(\neg\exists y(\neg R(y)) \;\Rightarrow\; \forall x(R(x)))$	elementary equivalence in 4 at 1 of hilb6 with hilb6
6	$(\neg\exists x(\neg R(x)) \;\Rightarrow\; \forall x(R(x)))$	rename y into x in 5 at occurence 1

□

Exchange of universal quantors:

Theorem 0.4.

$$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(\exists x(R(x,y))))$$

Proof.

1	$(\exists x(R(x)) \;\Rightarrow\; R(y))$	add axiom axiom5
2	$(\exists y(R(y)) \;\Rightarrow\; R(u))$	Substitute Variables in 1
3	$(\exists y(R(z,y)) \;\Rightarrow\; R(z,u))$	replace $R(@S_1)$ by $R(z,@S_1)$ in 2
4	$(\exists v(R(v)) \;\Rightarrow\; R(z))$	Substitute Variables in 1
5	$(\exists v(\exists w(R(v,w))) \;\Rightarrow\; \exists w(R(z,w)))$	replace $R(@S_1)$ by $\exists w(R(@S_1,w))$ in 4
6	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(R(z,y)))$	Substitute Variables in 5
7	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; R(z,u))$	HS with 6 and 3
8	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists z(R(z,u)))$	Generalize by z in 7
9	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists u(\exists z(R(z,u))))$	Generalize by u in 8
10	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(\exists z(R(z,y))))$	rename u into y in 9 at occurence 1
11	$(\exists x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(\exists x(R(x,y))))$	rename z into x in 10 at occurence 1

□

Implication of changing seqence of existence and universal quantor:

Theorem 0.5.

$$(\forall x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(\forall x(R(x,y))))$$

Proof.

1	$(\exists x(R(x)) \;\Rightarrow\; R(y))$	add axiom axiom5
2	$(\exists y(R(y)) \;\Rightarrow\; R(u))$	Substitute Variables in 1
3	$(\exists y(R(x,y)) \;\Rightarrow\; R(x,u))$	replace $R(@S_1)$ by $R(x,@S_1)$ in 2
4	$(R(y) \;\Rightarrow\; \forall x(R(x)))$	add axiom axiom6
5	$(R(x) \;\Rightarrow\; \forall z(R(z)))$	Substitute Variables in 4
6	$(R(x,u) \;\Rightarrow\; \forall z(R(z,u)))$	replace $R(@S_1)$ by $R(@S_1,u)$ in 5
7	$(\exists y(R(x,y)) \;\Rightarrow\; \forall z(R(z,u)))$	HS with 3 and 6
8	$(\forall x(\exists y(R(x,y))) \;\Rightarrow\; \forall z(R(z,u)))$	Particularize by x in 7
9	$(\forall x(\exists y(R(x,y))) \;\Rightarrow\; \exists u(\forall z(R(z,u))))$	Generalize by u in 8
10	$(\forall x(\exists y(R(x,y))) \;\Rightarrow\; \exists y(\forall x(R(x,y))))$	Substitute Variables in 9

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